

*Sally Dawson, BNL*  
*Hadronic Physics*  
*Maria Laach School, 2004*

- Introduction to the Standard Model
  - Review of the SU(2) x U(1) Electroweak theory
  - Experimental status of the EW theory
- The top quark and b quark
  - Basics
  - Review of Hadronic cross section calculations
- The Higgs boson
  - Why is it so crucial?
  - How can we find it?
- Beyond the SM at a hadron collider
  - Why are we sure there is physics BSM?
  - What will the LHC and Tevatron tell us?

## *Abelian Higgs Model*

- Why are the W and Z boson masses non-zero?
- U(1) gauge theory with single spin-1 gauge field,  $A_\mu$

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

- U(1) local gauge invariance:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x)$$

- Mass term for A:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

- Mass term violates local gauge invariance
- We understand why  $M_A = 0$

Gauge invariance is guiding principle

## Abelian Higgs Model, 2

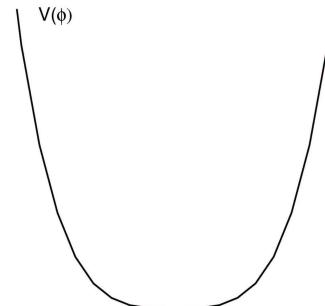
- Add complex scalar field,  $\phi$ , with charge  $-e$ :

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

- Where  $D_\mu = \partial_\mu - ieA_\mu$        $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$   
 $V(\phi) = \mu^2 |\phi|^2 + \lambda(|\phi|^2)^2$

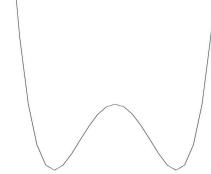
Most general potential  
invariant under  $\phi \rightarrow -\phi$

- $L$  is invariant under transformations:  $A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x)$   
 $\phi(x) \rightarrow e^{-ie\eta(x)} \phi(x)$
- Case 1:  $\mu^2 > 0$ 
  - QED with  $M_A=0$  and  $m_\phi=\mu$
  - Unique minimum at  $\phi=0$



## Abelian Higgs Model, 3

- Case 2:  $\mu^2 < 0$   $V(\phi) = -|\mu^2||\phi|^2 + \lambda(|\phi|^2)^2$
- Minimum energy state at:  $\langle\phi\rangle = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$
- Rewrite  $\phi \equiv \frac{1}{\sqrt{2}} e^{i\chi} (v + h)$
- L becomes:  $L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - ev A_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A^\mu A_\mu + \frac{1}{2} (\partial_\mu h \partial^\mu h + 2\mu^2 h^2) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \text{ interactions})$
- Theory now has:
  - Photon of mass  $M_A = ev$
  - Scalar field  $h$  with mass-squared  $-2\mu^2 > 0$
  - Massless scalar field  $\chi$
- What about mixed  $\chi$ -A propagator?
  - Remove by gauge transformation  $A'_\mu \equiv A_\mu - \frac{1}{ev} \partial_\mu \chi$
  - $\phi' \equiv e^{-i\chi/v} \phi = \frac{v+h}{\sqrt{2}}$
- $\chi$  field disappears
  - We say that it has been eaten to give the photon mass
  - $\chi$  field called Goldstone boson



Choosing a vacuum  
breaks U(1) symmetry

# *Higgs Mechanism summarized*

*Spontaneous breaking of a gauge theory  
by a non-zero VEV results in the disappearance  
of a Goldstone boson and its transformation into  
the longitudinal component of a massive gauge boson*

What about gauge invariance? Choice above called unitary gauge

- No  $\chi$  field
- Bad high energy behavior of  $A$  propagator:  $\Delta_{\mu\nu}(k) = -\frac{i}{k^2 - M_A^2} \left( g_{\mu\nu} - \frac{k^\mu k^\nu}{M_A^2} \right)$
- $R_\xi$  gauges more convenient:  $L_{GF} = - (1/2\xi)(\partial_\mu A^\mu + \xi ev\chi)^2$
- $\chi$  field is part of spectrum with  $M_\chi^2 = M_A^2$
- $\xi=1$ : Feynman gauge with massive  $\chi$
- $\xi=0$ : Landau gauge
  - $\chi$  massless with no coupling to physical Higgs boson

## *Aside on gauge boson counting*

- Massless photon has 2 transverse degrees of freedom
  - $\vec{p}_\gamma = (k, 0, 0, k)$
  - $\epsilon_\pm = (0, 1, \pm i, 0)/\sqrt{2}$
- Massive gauge boson has 3 degrees of freedom (2 transverse, 1 longitudinal)
  - $\epsilon_L = (|\vec{k}|, 0, 0, k_0)/M_V \rightarrow (k/M_V) + O(M_V/|\vec{k}|)$
- Count: Abelian Higgs Model
  - We start with: Massless gauge boson (2 dof), complex scalar (2 dof)
  - We end with: Massive gauge boson (3 dof), physical scalar (1 dof)

## Non-Abelian Higgs Mechanism

- Vector fields  $A_\mu^a(x)$  and scalar fields  $\phi_i(x)$

$$L = L_A + L_\phi, \quad L_\phi = \frac{1}{2} \sum_i (D^\mu \phi_i)^2 - V(\phi), \quad V(\phi) = \mu^2 \sum_i \phi_i^2 + \frac{\lambda}{2} \left( \sum_i \phi_i^2 \right)^2$$

- $L$  is invariant under the non-Abelian symmetry:

$$\phi_i \rightarrow (1 - \eta^a T^a)_{ij} \phi_j$$

- In exact analogy to the Abelian case

$$\begin{aligned} \frac{1}{2} (D^\mu \phi)^2 &\rightarrow \dots + \frac{1}{2} g^2 (T^a \phi)_i (T^b \phi)_i A_\mu^a A^{b\mu} + \dots \\ &\xrightarrow{\phi \rightarrow \phi_0} \dots + \frac{1}{2} g^2 (T^a \phi_0)_i (T^b \phi_0)_i A_\mu^a A^{b\mu} + \dots \end{aligned}$$

- $T^a \phi \neq 0 \Rightarrow$  Massive vector boson + Goldstone boson
- $T^a \phi = 0 \Rightarrow$  Massless vector boson + massive scalar field

# Goldstone theorem in general

- When a continuous symmetry is spontaneously broken (ie, it is not a symmetry of the physical vacuum), the theory has one massless scalar particle for each broken generator

- Consider L with  $N_G$  real scalar fields  $\phi$

$$L = L_A + L_\phi, \quad L_\phi = \frac{1}{2} \sum_i (D^\mu \phi_i)^2 - V(\phi), \quad V(\phi) = \mu^2 \sum_i \phi_i^2 + \frac{\lambda}{2} \left( \sum_i \phi_i^2 \right)^2$$

- G is a symmetry group such that

$$\delta\phi_i = -\eta^a T^a{}_{ij} \phi_j$$

- Since the potential is invariant under G

$$\delta V = \frac{\partial V}{\partial \phi_i} \delta\phi_i = -i \frac{\partial V}{\partial \phi_i} \eta^a (T^a)_{ij} \phi_j = 0$$

- Gauge parameters are arbitrary, giving  $N_G$  equations:

$$\frac{\partial V}{\partial \phi_i} (T^a)_{ij} \phi_j = 0 \quad \text{for } a = 1 \dots N_G$$

- Taking the second derivative:

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_j} (T^a)_{ik} \phi_k + \frac{\partial V}{\partial \phi_i} (T^a)_{ij} = 0 \quad \text{for } a = 1 \dots N_G$$



Vanishes at minimum

## More on Goldstone Theorem

- At minimum of potential,  $\phi = \phi_0$

$$\left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\phi=\phi_0} (T^a)_{ik} \phi_{0k} = 0 \quad \text{for } a = 1 \dots N_G$$

Scalar mass matrix

- After choosing ground state, a subgroup  $g$  of  $G$  with dimension  $n_g$ , remains symmetry of vacuum. For generators of  $g$

$$(T^a)_{ik} \phi_{k0} = 0 \quad \text{for } a = 1 \dots n_g$$

- For  $(N_G - n_g)$  generators that break the symmetry

$$(T^a)_{ik} \phi_{k0} \neq 0 \quad \text{for } a = n_g + 1 \dots N_G$$

- $(N_G - n_g)$  zero eigenvalues of the mass matrix: the massless Goldstone bosons

Useful result for general model building

## *SM Higgs Mechanism*

- Standard Model includes complex Higgs SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

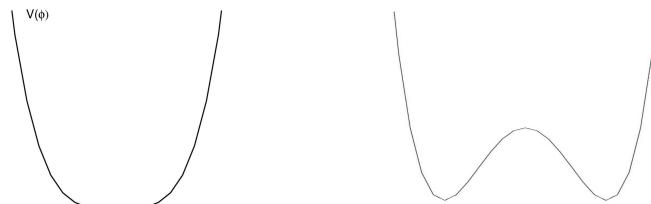
- With SU(2) x U(1) invariant scalar potential

$$V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad \text{Invariant under } \Phi \rightarrow -\Phi$$

- If  $\mu^2 < 0$ , then spontaneous symmetry breaking

- Minimum of potential at:  $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

- Choice of minimum breaks gauge symmetry
  - Why is  $\mu^2 < 0$ ?



Aside:  $\mu^2 < 0$  question is a motivation for mSUGRA

## More on SM Higgs Mechanism

- Couple  $\Phi$  to  $SU(2) \times U(1)$  gauge bosons ( $W_i^\mu$ ,  $i=1,2,3$ ;  $B^\mu$ )

$$L_S = (D^\mu \Phi)^+ (D^\mu \Phi) - V(\Phi)$$

$$D_\mu = \partial_\mu + i \frac{g}{2} \sigma^i W^i_\mu + i \frac{g'}{2} B_\mu$$

- Gauge boson mass terms from:

$$(D_\mu \phi)^+ D^\mu \phi \rightarrow \dots + \frac{1}{8} (0, v) (g W_\mu^a \sigma^a + g' B_\mu) (g W^{b\mu} \sigma^b + g' B^\mu) \begin{pmatrix} 0 \\ v \end{pmatrix} + \dots$$

$$\rightarrow \dots + \frac{v^2}{8} (g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-g W_\mu^3 + g' B_\mu)^2) + \dots$$

- With massive gauge bosons:

$$W_\mu^\pm = (W_\mu^1 \pm W_\mu^2) / \sqrt{2}$$

$$Z_\mu^0 = (g W_\mu^3 - g' B_\mu) / \sqrt{(g^2 + g'^2)}$$

$$M_W = gv/2$$

$$M_Z = \sqrt{(g^2 + g'^2)}v/2$$

- Orthogonal combination to  $Z$  is massless photon

$$A_\mu^0 = (g' W_\mu^3 + g B_\mu) / \sqrt{(g^2 + g'^2)}$$

$$M_W = M_Z \cos \theta_W$$

- Weak mixing angle defined

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

## *SM Higgs Mechanism continued*

- Photon corresponds to electric charge  $Q_{em} = (T_3 + Y)/2$

- Electric charge of vacuum is zero

$$Q_{em} \langle \Phi \rangle_0 = \frac{1}{2} \left[ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = 0$$

- SM has special relationship (at tree level)

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}$$

- In theories with Higgs bosons in general representations:

$$\rho = \frac{\sum_i [4\tau_i(\tau_i+1) - (Y_i)^2] v_i^2}{2 \sum_i (Y_i)^2 v_i^2}$$

- SU(2) Higgs doublets,  $\tau=1/2$ ,  $Y=1 \Rightarrow \rho=1$
- SU(2) singlets don't contribute
- Other representations require tuning: eg triplet Higgs  $\tau=3$  and  $Y=4$

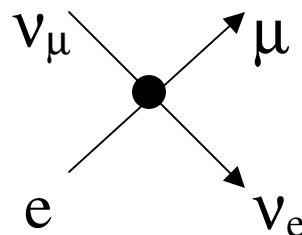
Significant restriction on model building

## *More on SM Higgs Mechanism*

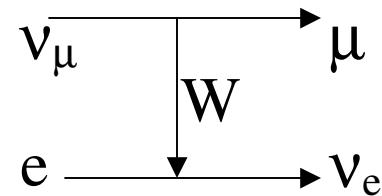
- Generate mass for W,Z using Higgs mechanism
  - Higgs VEV breaks  $SU(2) \times U(1) \rightarrow U(1)_{\text{em}}$
  - Single Higgs doublet is minimal case
- Just like Abelian Higgs model
  - Goldstone Bosons  $\Phi = \frac{1}{\sqrt{2}} e^{-i \frac{\bar{\omega}_i \vec{\sigma}_i}{v}}$        $\omega_i \rightarrow \omega^\pm, z$
- Before spontaneous symmetry breaking:
  - Massless  $W_i$  ( $i=1,2,3$ ), B, Complex  $\Phi$
- After spontaneous symmetry breaking:
  - Massive  $W^\pm, Z$ ; massless  $\gamma$ ; physical Higgs boson  $h$

## Higgs VEV fixed

- Consider  $\nu_\mu e \rightarrow \mu \nu_e$



- EW Theory:



$$-i2\sqrt{2}G_\mu g_{\mu\nu}\bar{u}_\mu\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_{\nu_\mu}\bar{u}_{\nu_e}\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)u_e$$

$$\frac{ig^2}{2}\frac{1}{k^2-M_W^2}g_{\mu\nu}\bar{u}_\mu\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_{\nu_\mu}\bar{u}_{\nu_e}\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)u_e$$

For  $|k| \ll M_W$ ,  $2\sqrt{2}G_\mu = g^2/2M_W^2$

For  $|k| \gg M_W$ ,  $\sigma \sim 1/E^2$

## *Adding fermions is more of same...*

- Include color triplet quark doublet     $Q_L = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$        $i=1,2,3$ 
  - Right handed quarks are SU(2) singlets,  $u_R = (1+\gamma_5)u$ ,  $d_R = (1-\gamma_5)d$
- With weak hypercharge
  - $Y_{uR} = 4/3$ ,  $Y_{dR} = -2/3$ ,  $Y_{QL} = 1/3$
- Couplings of charged current to W and Z's take the form:

$$L_{Wqq} = -\frac{g}{2\sqrt{2}} (\bar{u} \gamma^\mu (1-\gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1-\gamma_5) u W_\mu^-) \quad \leftarrow \quad \boxed{Q_{em} = (T_3 + Y)/2}$$

$$L_{Zqq} = -\frac{g}{4 \cos \theta_W} \bar{q} \gamma^\mu [L_q (1-\gamma_5) + R_q (1+\gamma_5)] q Z_\mu$$

$L_q = T_3 + 2Q_{em} \sin^2 \theta_W$   
 $R_q = 2Q_{em} \sin^2 \theta_W$

- More than one quark doublet implies flavor mixing (see lecture 2)
  - Z decays measured precisely at LEP

## Higgs Parameters

- $G_\mu$  measured precisely

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} \quad v^2 = (\sqrt{2}G_\mu)^{-1} = (246 GeV)^2$$

- Higgs potential has 2 free parameters,  $\mu^2, \lambda$
- Trade  $\mu^2, \lambda$  for  $v^2, M_h^2$

$$V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

- Large  $M_h \rightarrow$  strong Higgs self-coupling
- A priori, Higgs mass can be anything

$$V = \frac{M_h^2}{2} h^2 + \frac{M_h^2}{2v} h^3 + \frac{M_h^2}{8v^2} h^4$$

$$v^2 = -\frac{\mu^2}{2\lambda}$$
$$M_h^2 = 2v^2\lambda$$

## *Parameters of $SU(2) \times U(1)$ Sector*

- $g, g', v, M_h \Rightarrow$  Trade for:
  - $\alpha = 1/137.03599911(46)$  from  $(g-2)_e$  and quantum Hall effect
  - $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$  from muon lifetime
  - $M_Z = 91.1876 \pm 0.0021 \text{ GeV}$
  - Plus quark and Higgs masses

Everything predicted in terms of  $\alpha, G_F, M_Z$  and masses

This is a theory we can test!

Predict  $M_W, \sin\theta_W$

## Now include Leptons

- Simplest case, include an SU(2) doublet of left-handed leptons

$$\Psi_L = \begin{pmatrix} \nu_L = \frac{1}{2}(1-\gamma_5)\nu \\ e_L = \frac{1}{2}(1-\gamma_5)e \end{pmatrix}$$

- Only right-handed electron,  $e_R = (1+\gamma_5)e/2$

– No right-handed neutrino

- Define weak hypercharge,  $Y$ , such that  $Q_{em} = (T_3 + Y)/2$

$$T_3 = \pm 1$$

–  $Y_L = -1$

–  $Y_R = -2$

- By construction Isospin,  $T_3$ , commutes with weak hypercharge  
 $[T_3, Y] = 0$

- Couple gauge fields to leptons

$$L_{leptons} = \bar{e}_R i \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} Y B_\mu \right) e_R + \bar{\Psi}_L i \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} Y B_\mu + i \frac{g}{2} \sigma_i W_i^\mu \right) \Psi_L$$

## *After spontaneous symmetry breaking....*

- Couplings to leptons fixed:

$$\bar{v}eW^{+\mu} : -i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

$$\bar{v}vZ^\mu : -i\frac{g}{4}\gamma^\mu(1-\gamma_5)$$

$$\bar{e}eZ^\mu : -i\frac{g}{4\cos\theta_W}\gamma^\mu[R_e(1+\gamma_5)+L_e(1-\gamma_5)]$$

$$L_e = T_3 + 2\sin^2\theta_W$$

$$R_e = 2\sin^2\theta_W$$

- Trivially calculate decay widths from:

$$\Gamma(V \rightarrow f\bar{f}) = \frac{1}{2M_V} |A|^2 \frac{1}{8\pi}$$

$$\Gamma(Z \rightarrow \nu\bar{\nu}) = \frac{G_F M_Z^2}{12\pi\sqrt{2}}$$

$$\Gamma(Z \rightarrow e^+e^-) = \frac{G_F M_Z^2}{12\pi\sqrt{2}} (R_e^2 + L_e^2)$$

## Gauge boson self-interactions

- Yang-Mills for gauge fields:

$$L_{YM} = -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}$$

$$\begin{aligned} W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc}W_\mu^b W_\nu^c \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned}$$

- In terms of physical fields:

$$\begin{aligned} L_{YM} = & -\frac{1}{4}\left|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)\right|^2 \\ & -\frac{1}{4}\left|\partial_\mu Z_\nu - \partial_\nu Z_\mu + ie\frac{c_W}{s_W}(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)\right|^2 \\ & -\frac{1}{2}\left|\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu) + ie\frac{c_W}{s_W}(W_\mu^+ Z_\nu - W_\nu^+ Z_\mu)\right|^2 \end{aligned}$$

- Triple and quartic gauge boson couplings fundamental prediction of model

# Three Gauge Boson vertices fixed

Example:  $v\bar{v} \rightarrow W^+W^-$

➤ t-channel amplitude:

$$A_t(v\bar{v} \rightarrow W^+W^-) = -i \frac{g^2}{8} \bar{v}(q) \gamma^\mu (1-\gamma_5) \frac{k}{k^2} \gamma^\nu (1-\gamma_5) u(p) \epsilon_\mu(p^-) \epsilon_\nu(p^+)$$

➤ In center-of-mass frame:

$$p = \frac{\sqrt{s}}{2} (1, 0, 0, 1)$$

$$q = \frac{\sqrt{s}}{2} (1, 0, 0, -1)$$

$$p^+ = \frac{\sqrt{s}}{2} (1, 0, \beta_w \sin \theta, \beta_w \cos \theta)$$

$$p^- = \frac{\sqrt{s}}{2} (1, 0, -\beta_w \sin \theta, -\beta_w \cos \theta)$$

➤ Interesting physics is in the longitudinal W sector:  $\epsilon^+ \rightarrow \frac{p^+}{M_W}$

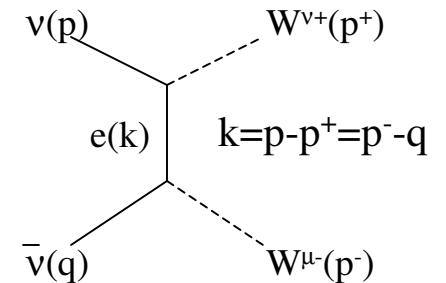
$$A_t(v\bar{v} \rightarrow W_L^+ W_L^-) = -i \frac{g^2}{8M_W^2} \bar{v}(q) p^- (1-\gamma_5) \frac{k}{k^2} p^+ (1-\gamma_5) u(p)$$

➤ Use Dirac Equation:  $\not{p} u(p) = 0$

$$A_t(v\bar{v} \rightarrow W_L^+ W_L^-) = i \frac{g^2}{4M_W^2} \bar{v}(q) k (1+\gamma_5) u(p)$$



$$\left| A_t(v\bar{v} \rightarrow W_L^+ W_L^-) \right|^2 = 2G_F^2 s^2 \beta_w^2 \sin^2 \theta$$



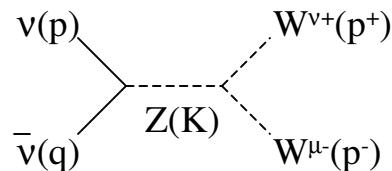
## $\nu\bar{\nu} \rightarrow W^+W^-$ continued

- SM has additional contribution from s-channel Z exchange

$$A_s(\nu\bar{\nu} \rightarrow W^+W^-) = -i \frac{g^2}{4(s-M_Z^2)} \bar{v}(q)\gamma_\mu(1-\gamma_5)u(p) \left( g^{\mu\nu} - \frac{K^\mu K^\nu}{M_Z^2} \right) [g_{\lambda\rho}(p^- - p^+)_\nu + g_{\lambda\nu}(p^+ + K)_\rho - g_{\rho\nu}(p^- + K)_\lambda] \epsilon^\lambda(p^+) \epsilon^\rho(p^-)$$

- For longitudinal W's

$$A_s(\nu\bar{\nu} \rightarrow W_L^+W_L^-) = i \frac{g^2}{4M_W^2} \bar{v}(q)(p^+ - p^-)(1-\gamma_5)u(p)$$



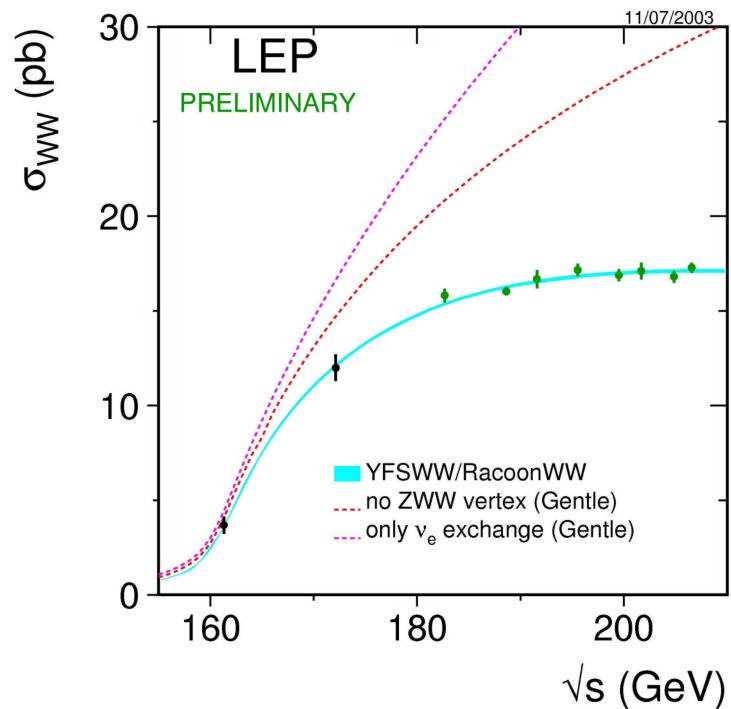
$$A_s(\nu\bar{\nu} \rightarrow W_L^+W_L^-) = -i \frac{g^2}{4M_W^2} \bar{v}(q)k(1+\gamma_5)u(p)$$

Contributions which grow with energy cancel between t- and s-channel diagrams

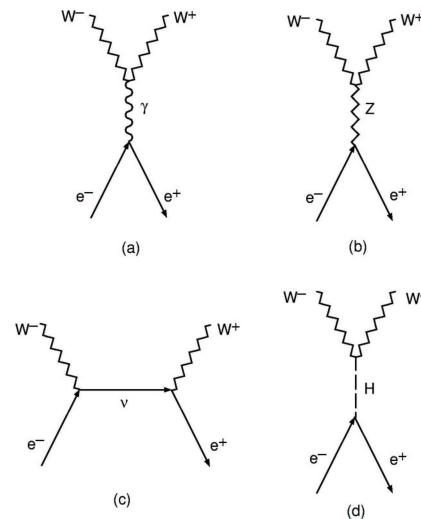


Depends on 3-gauge boson couplings

# No deviations from SM at LEP2



No evidence for Non-SM 3  
gauge boson vertices



LEP EWWG, hep-ex/0312023

Just starting to measure gauge  
boson pair production at the  
Tevatron

## Now Include Quarks

- $Y_{QL} = 1/3, Y_{uR} = 4/3, Y_{dR} = -2/3$
- This fixes charged and neutral currents

$$L_w = -\frac{g}{2\sqrt{2}} [\bar{u} \gamma^\mu (1 - \gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u W_\mu^-]$$

$$L_Z = \frac{ig}{4 \cos \theta_W} \gamma^\mu \bar{q} (R_q (1 + \gamma_5) + L_q (1 - \gamma_5)) q Z_\mu$$

$$L_q = T_3 - 2 Q_{em} \sin^2 \theta_W$$

$$R_q = -2 Q_{em} \sin^2 \theta_W$$

Three generations of quarks:  $\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$        $u_R, d_R, c_R, s_R, t_R, b_R$

- *Mass eigenstates are not gauge eigenstates*

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

V is unitary

- *This mixes generations in charged currents*
- *Note that Z couplings remain diagonal in flavor space*

## ***SM also accommodates fermion masses***

- Scalar couplings to fermions:  $L_d = -\lambda_d \bar{Q}_L \Phi d_R + h.c.$ 
  - with left handed SU(2) fermion doublets:  $\mathcal{Q}_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$

- Effective Higgs-fermion coupling

$$-\lambda_d \frac{1}{\sqrt{2}} (\bar{u}_L, \bar{d}_L) \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R + h.c.$$

- Mass term for down quark:

$$\lambda_d = -\frac{M_d \sqrt{2}}{v}$$

- $M_u$  from  $\Phi_c = i\tau_2 \Phi^*$  (not allowed in SUSY)
- For 3 generations,  $i,j=1,2,3$  (flavor indices)

$$L_Y = -\frac{(v+h)}{\sqrt{2}} \sum_{ij} (\lambda_u^{ij} \bar{u}_L^i u_R^j + \lambda_d^{ij} \bar{d}_L^i d_R^j) + h.c.$$

- Unitary matrices diagonalize mass matrices

$$\begin{aligned} u_L^i &= U_u^{ij} u_L^{mj} & d_L^i &= U_d^{ij} d_L^{mj} \\ u_R^i &= V_u^{ij} u_R^{mj} & d_R^i &= V_d^{ij} d_R^{mj} \end{aligned}$$

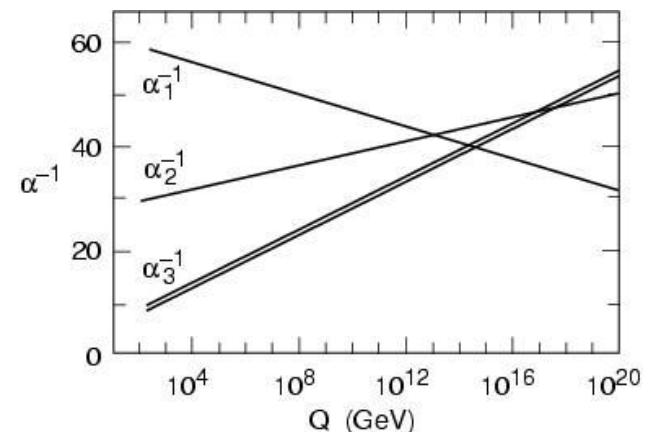
- Yukawa couplings are *diagonal* in mass basis
- Neutral currents remain flavor diagonal
- Not necessarily true in models with extended Higgs sectors

# Standard Model in a Nutshell

- $SU(3) \times SU(2) \times U(1)$  gauge theory
  - Couplings  $g_3, g, g'$
- 3 generations of quarks and leptons

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$u_R, d_R, c_R, s_R, t_R, b_R$$
$$e_R, \mu_R, \tau_R$$



Input  $g$  and  $g'$ . They meet at  $10^{15}$  GeV and predict  $\alpha_s(M_Z)=0.073\pm.002$

No unification in SM

# Precision measurements constrain SM

- Let's not be blasé
- This is an amazing plot!
  - $M_Z$ ,  $G_F$ ,  $\alpha(M_Z)$ ,  $M_W$ ,  $M_f$ ,  $\alpha_s(M_Z)$
- Perturbatively calculate observables in terms of well measured parameters

Low  $Q^2$  data not included in fit



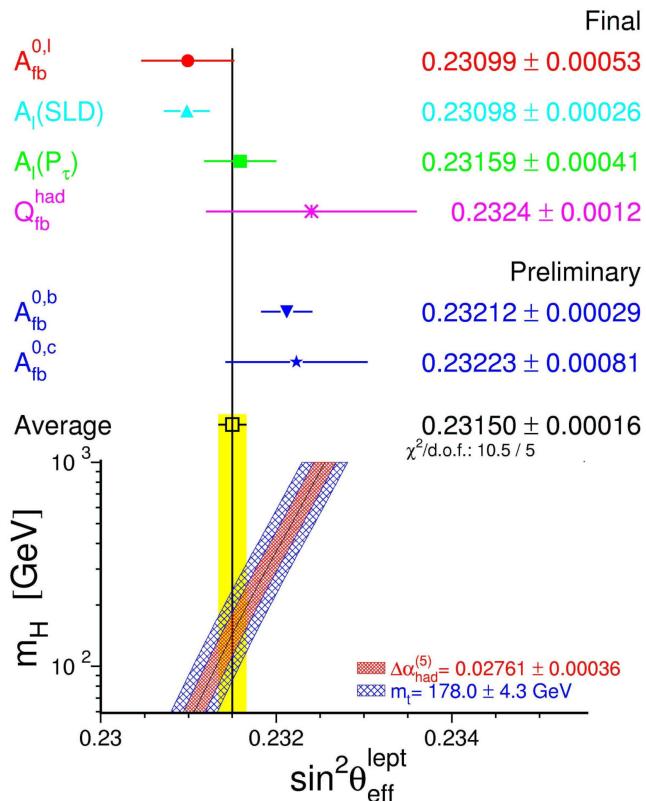
## *One Loop Corrections to SM*

- At tree level  $\sin^2 \theta_w = 1 - \frac{M_w^2}{M_Z^2}$
- Beyond tree level, many possible definitions
- Include vertex and fermion self-energy radiative corrections in effective Z couplings:

$$-i \frac{g}{2 \cos \theta_w} \bar{\Psi}_f \gamma^\mu (g_V^f - g_A^f) \Psi_f$$

- Define  $\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \frac{g_V^l}{g_A^l} \right)$

# *Hard to explain $\sin^2 \theta_{\text{eff}}^{\text{lept}}$* (not a new puzzle)



- Measurements from leptons and hadrons tend to disagree
- 2 most precise measurements differ by  $2.9\sigma$

Modify Zbb vertex?

Hard to do consistently

$$A_{\text{fb}}^{0\text{b}} = (3/4) A_e A_b$$

## *Interpreting $\sin^2 \theta_{\text{eff}}^{\text{lept}}$*

➤  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$  from leptons:

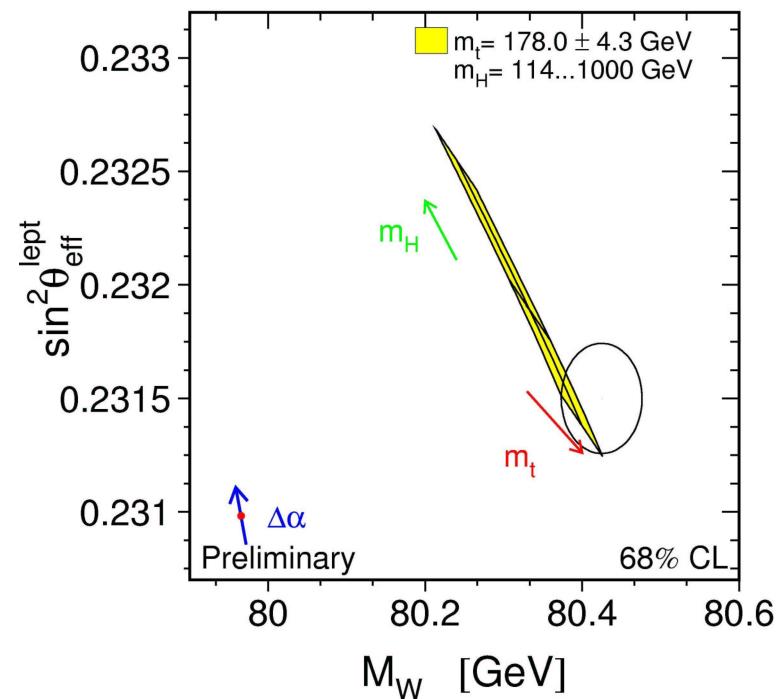
- $A_l(\text{SLD}) \Rightarrow .23098 \pm 0.00026$

- $M_h$  conflicts with direct Higgs search limit

➤  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$  from hadrons:

- $A_{fb}^{0,b}(\text{LEP}) \Rightarrow .23212 \pm 0.00029$

- Conflicts with  $M_W$  measurement

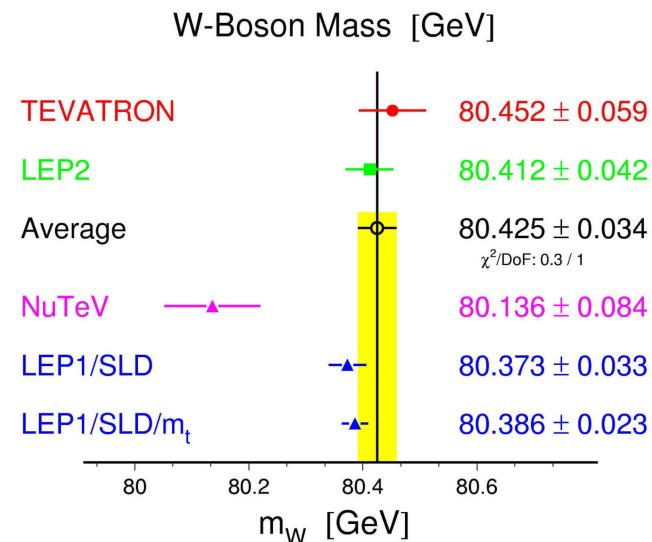
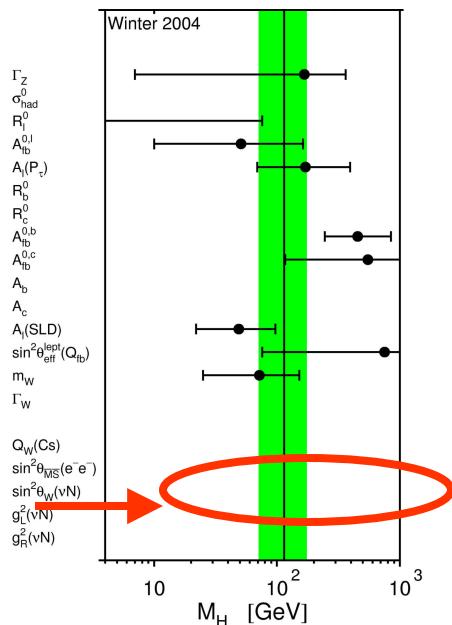


**LEPP EWWG 04**

# *M<sub>W</sub>(exp) is a little high?*

- Fit precision measurements (include M<sub>t</sub>)
  - M<sub>W</sub>(fit)=80.386±.023 GeV

LEP EWWG 2004



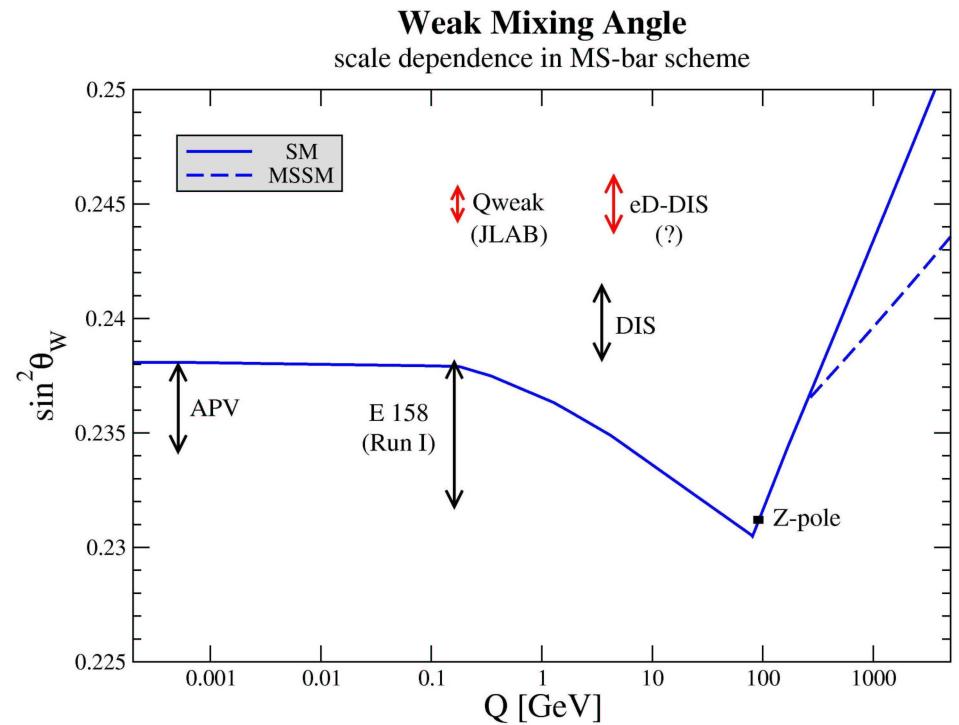
$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W} \frac{1}{1 - \Delta r(M_t, M_h)}$$

World Average 2003:  
M<sub>W</sub>=80.426 ± .034 GeV

# *What about low $Q^2$ data?*

- Moller Scattering
- NuTeV
- Atomic Parity Violation
- $(g-2)_\mu$

*Low  $Q^2$  data tests  
understanding of  
scale dependence*



Erler and Ramsey-Musolf, hep-ph/0404291

## NuTeV

- NuTeV

$$\sin^2 \theta_W = 0.2277 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst})$$
$$-0.00022 \frac{M_t^2 - (175\text{GeV})^2}{(50\text{GeV})^2} + 0.00032 \ln\left(\frac{M_h}{150\text{GeV}}\right)$$

- Global fit:

*3 $\sigma$  discrepancy?*

$$\sin^2 \theta_W = 0.2227 \pm 0.0004$$

- Understanding of theory error critical

*Experimental Observables*



*Theory Calculation*

# More NuTeV

- Intense theoretical effort
- NuTeV analysis related to Paschos-Wolfenstein ratio:

$$R^- = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} \approx \frac{1}{2} - \sin^2 \theta_W + \delta R_A^- + \delta R_{QCD}^- + \delta R_{EW}^-$$

- Non-isoscalar target

$$\delta R_s^- \approx -\left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_W\right) \frac{[S^-]}{[Q^-]}$$

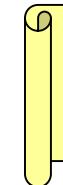
- Positive  $[S^-]$  goes towards removing discrepancy

$$[S^-] \equiv \int x(s(x) - \bar{s}(x)) dx$$

- CTEQ fit to  $[S^-]=0.002$  explains  $1.5\sigma$  of discrepancy
- CTEQ:  $-.005 < \delta \sin^2 \theta_W < .004$

- Higher order  $O(\alpha)$  electroweak corrections

- Apparent discrepancy with older results used by NuTeV
- Factorization scheme dependence and treatment of final state photons can account for  $3\sigma$  difference

 Only Experimentalists  
can tell for sure

Kretzer, hep-ph/0405221

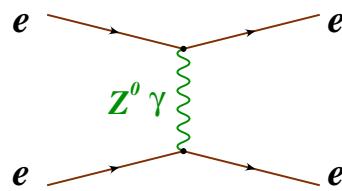
Diener, Dittmaier, Hollik, hep-ph/0310364

# *Atomic Parity Violation*

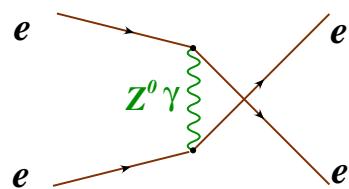
- 2002 fit showed at  $1.5\sigma$  deviation for atomic parity violation (weak charge of Cesium nucleus)
- New calculation of QED corrections
- $Q_w(\text{exp}) = -72.84 \pm 0.29(\text{exp}) \pm .36(\text{th})$
- Good agreement with best fit value
- $Q_w(\text{fit}) = -72.880 \pm 0.003$

# Moller Scattering, E158

Purely leptonic reaction



$$g_{ee} \sim 1 - 4\sin^2\theta_W$$



$$A_{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \cdot \frac{1-y}{1+y^4 + (1-y)^4} \cdot F_{brem} \cdot (1 - 4\sin^2\theta_W^{eff})$$

Run I + Run II :

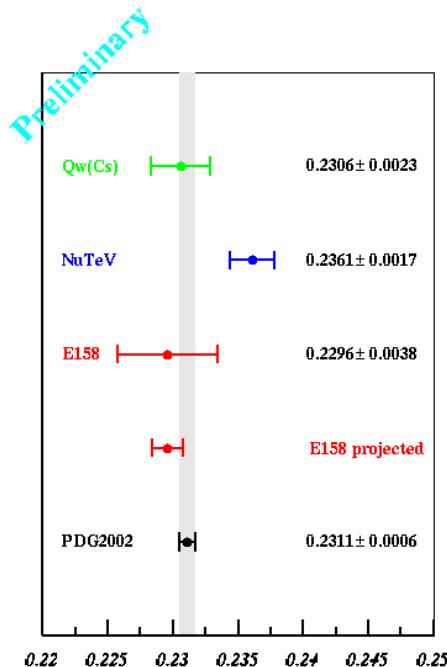
$$\sin^2\theta_W(Q^2=0.026 \text{ GeV}^2) =$$

$$0.2379 \pm 0.0016 \text{ (stat)} \pm 0.0013 \text{ (syst)}$$

Theory:

$$\sin^2\theta_W(Q^2=0.026 \text{ GeV}^2) = 0.2385 \pm 0.0006 \text{ (theory)}$$

*Extrapolate to  $M_Z$*

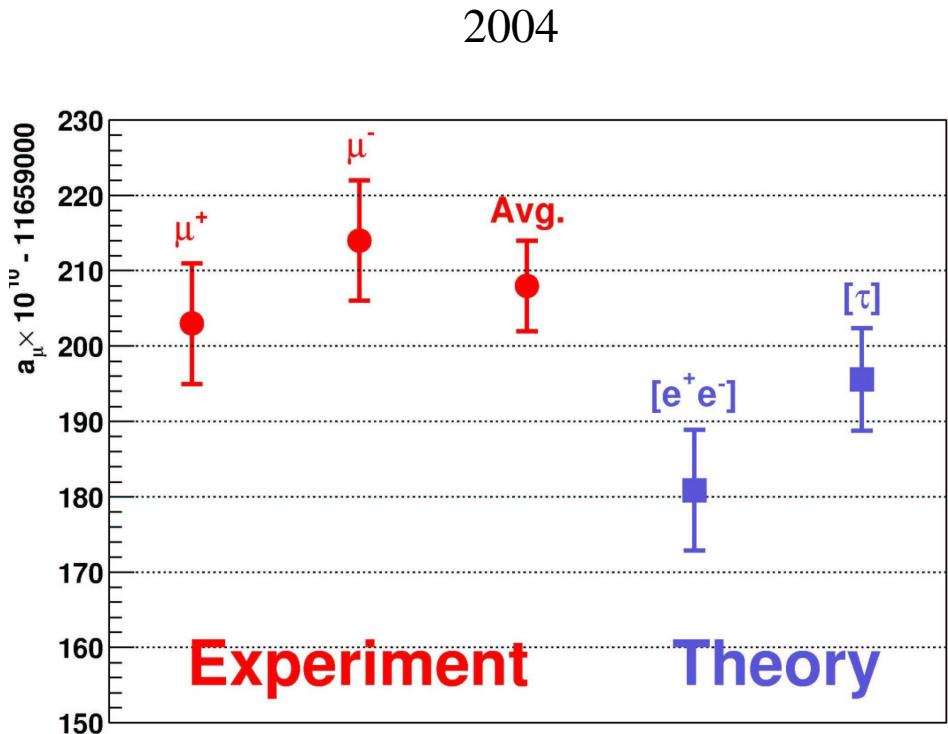
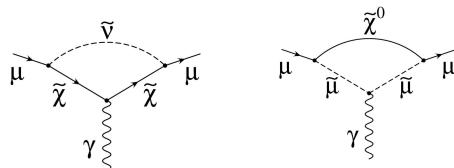


## (g-2) $_{\mu}$

- Stimulated new theoretical efforts (SM and beyond)
- No evidence for CPT violation
- Progress in understanding hadronic contributions
- Naturally explained by SUSY

$$\delta a_{\mu} \approx 150 \times 10^{-11} \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \tan \beta$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} \approx 300 \times 10^{-11}$$



- $e^+e^-$  data:  $2.7\sigma$  effect
- $\tau$  data:  $1.4\sigma$  effect

# Contributions to $(g-2)_\mu$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}} + a_\mu^{\text{weak}}$$

= (QED)	$(11\ 658\ 470.35 \pm 0.28)10^{-10}$ (5-loop!)
+ (had,LO)	$(684.7 \text{ to } 709.0 \pm 6)10^{-10}$ (Big spread, largest error)
+ (had,HO)	$(-10.0 \pm 0.6)10^{-10}$
+ (had,LBL)	$(8.0 \pm 4.0)10^{-10}$ (sign change since 1998)
+ (weak)	$(15.4 \pm 0.2)10^{-10}$ (2-loop)

$a_\mu^{\text{had,LO}}$  from data via dispersion integral

$$a_\mu^{\text{had,LO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty \sigma_{\text{had}}^0(s) K(s) ds$$

Recent data included CMD-2,  
SND, BES 2-5 GeV, ALEPH  $\tau$ .  
NEW: CMD-2 prelim update

Dominated by low energy region,  $\rho$  resonance

P. Gambino, Summer 2003

# *Hadronic Contributions*

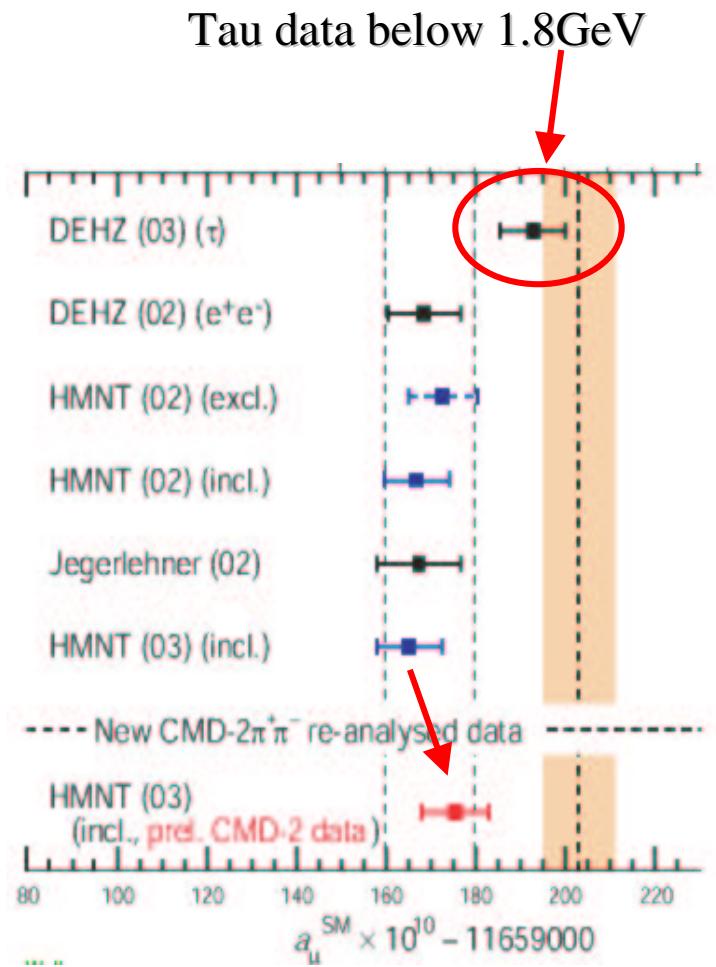
Final CMD-2  $\pi\pi$  data (2002) 0.6% syst error!

Hagiwara et al with  $e^+e^-$  data:  
 $a_\mu^{\text{had,LO}} = 691.7 \pm 5.8_{\text{exp}} \pm 2.0_{\text{r.c.}}$

This translates to a  $\sim 2\text{-}2.5\sigma$  discrepancy

Using  $\tau$  data below 1.8 GeV Davier et al:  
 $a_\mu^{\text{had,LO}} = 709.0 \pm 5.1_{\text{exp}} \pm 1.2_{\text{r.c.}} \pm 2.8_{\text{SU(2)}}$

$\tau$  data consistent with SM



Hagiwara et al, hep-ph/0312250

Davier et al, hep-ex/0312065

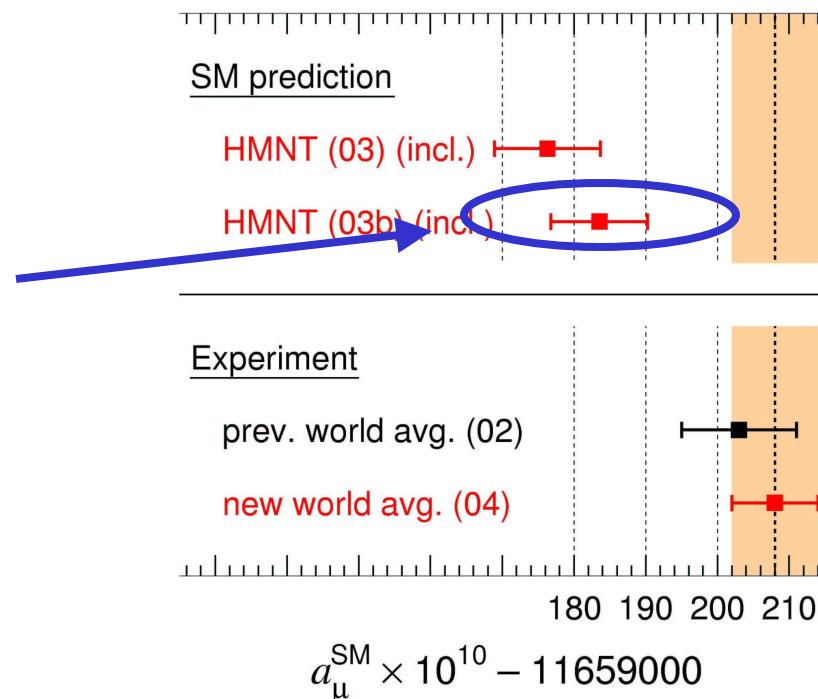
# *More news on hadronic contributions (Spring, 2004)*

## ➤ *Updated light by light*

- Understanding of chiral logs
- $\delta a_\mu = 56 \times 10^{-11}$

## ➤ *Updated QED*

- Coefficient of  $(\alpha/\pi)^4$  changed
- $\delta a_\mu = 13.7 \times 10^{-11}$



Melnikov & Vainshtein, hep-ph/0312226

Kinoshita & Nio, hep-ph/0402206

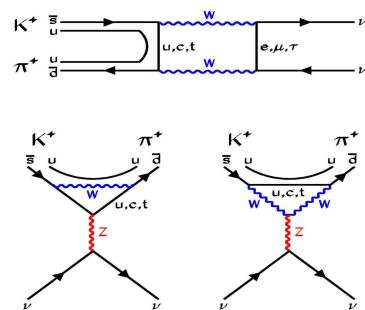
$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

BNL E949: 3 events

$$\text{BR(exp)} = 1.47^{+1.30}_{-0.89} \times 10^{-10}$$

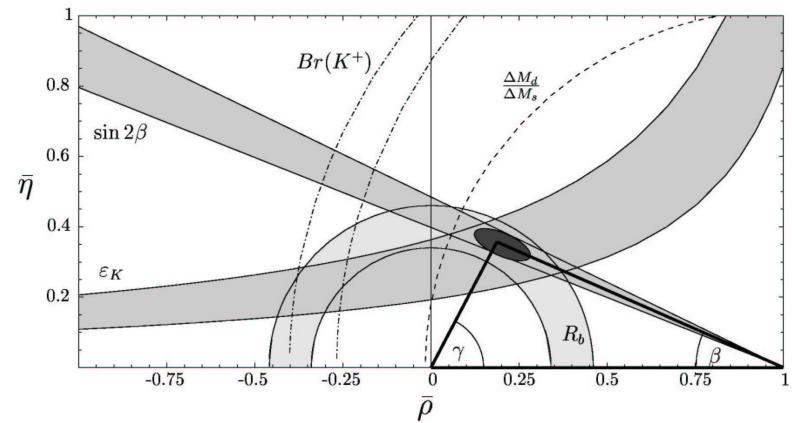
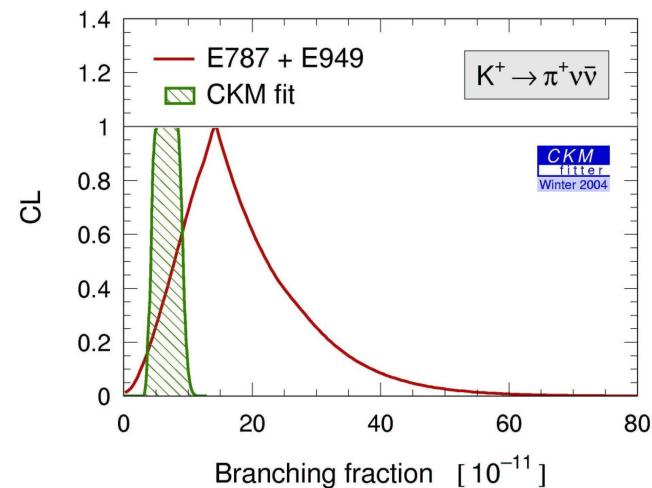
SM prediction updated with new  $M_t$

$$\text{BR(th)} = 7.8 \pm 1.2 \times 10^{-11}$$



Buras, Schwab, Uhlig, hep-ph/0405132

E949, hep-ph/0403036



*Electroweak physics is in even  
better shape this year than last  
year....*

*Tevatron is becoming our next precision  
tool for QCD and EW physics*